

$$Z_t - Z_b = \frac{R\overline{T}_v}{g_0} \ln \left( \frac{p_b}{p_t} \right), \quad (3.1)$$

$$p_h = \int_h^\infty g \rho dz. \quad (3.2)$$

$$\frac{\partial p_s}{\partial t} \approx - \int_0^{p_s} \nabla_p \cdot \mathbf{V} dp, \quad (3.3)$$

$$\frac{\delta p_s}{\delta t} = \frac{\partial p_s}{\partial t} + \mathbf{C} \cdot \nabla_z p_s \quad (3.4)$$

$$\frac{\partial p_s}{\partial t} = -\mathbf{C} \cdot \nabla_z p_s. \quad (3.5)$$

$$\begin{aligned} \left( \nabla_p^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \right) \omega &= \frac{f_0}{\sigma} \frac{\partial}{\partial p} \left( \mathbf{V}_g \cdot \nabla_p \left( \frac{\nabla_p^2 \Phi}{f_0} + f \right) \right) \\ &+ \frac{1}{\sigma} \nabla_p^2 \left( \mathbf{V}_g \cdot \nabla_p \left( -\frac{\partial \Phi}{\partial p} \right) \right) \\ &- \frac{1}{\sigma} \nabla_p^2 \left( \frac{\dot{Q}}{\rho c_p} \right) \\ &- \frac{f_0}{\sigma} \frac{\partial}{\partial p} (\mathbf{k} \cdot \nabla_p \times \mathbf{F}). \end{aligned} \quad (3.6)$$

$$\omega(x, y, p) = \omega_0 \sin \left( \frac{2\pi x}{L} \right) \sin \left( \frac{2\pi y}{L} \right) \left( \frac{(p_0 - p)(p - p_t)}{(\frac{1}{2})^2 (p_0 - p_t)^2} \right) \quad (3.7)$$

$$\left( \nabla_p^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \right) \omega = -(\omega_v + \omega_T + \omega_H + \omega_f) \quad (3.8)$$

$$\omega_v = \frac{f_0}{\sigma} \frac{\partial}{\partial p} \left( -\mathbf{V}_g \cdot \nabla_p \left( \frac{\nabla_p^2 \Phi}{f_0} + f \right) \right) = \frac{f_0}{\sigma} \frac{\partial}{\partial p} (-\mathbf{V}_g \cdot \nabla_p \eta_g). \quad (3.13)$$

$$\omega_T = \frac{-1}{\sigma} \nabla_p^2 \left( \mathbf{V}_g \cdot \nabla_p \left( -\frac{\partial \Phi}{\partial p} \right) \right) = \frac{R}{\sigma p} \nabla_p^2 (-\mathbf{V}_g \cdot \nabla_p T) \quad (3.17)$$

$$\left( \nabla_p^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \right) \omega = \frac{2}{\sigma} \frac{\partial \mathbf{V}_g}{\partial p} \cdot \nabla_p \eta_g. \quad (3.19)$$

$$\left( \nabla_p^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \right) \frac{\partial \Phi}{\partial t} = -f_0 \mathbf{V}_g \cdot \nabla_p \eta_g + \frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left( \mathbf{V}_g \cdot \nabla_p \left( -\frac{\partial \Phi}{\partial p} \right) \right) - \frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left( \frac{\dot{Q}}{\rho c_p} \right) \quad (3.20)$$

$$\sigma = \frac{1}{\theta} \frac{\partial \Phi}{\partial p} \frac{\partial \theta}{\partial p} = g \frac{\partial z}{\partial p} \left( \frac{1}{T} \frac{\partial T}{\partial p} - \frac{\kappa}{p} \frac{\partial p}{\partial p} \right) = \frac{R}{p} \left( \Gamma_d - \frac{\partial T}{\partial p} \right) \quad (3.21)$$

$$\left( -2 \left( \frac{2\pi}{L} \right)^2 \sigma + \left( \frac{-8f_0^2}{(p_0 - p_T)^2} \right) \right) \omega_d = -forcing = -\sigma(\omega_v + \omega_T + \omega_H + \omega_f) \quad (3.25)$$

which can be written in shorthand as

$$(A_d + B)\omega_d = +forcing \quad (3.26)$$

$$A_d = 2 \left( \frac{2\pi}{L} \right)^2 \sigma = 2 \left( \frac{2\pi}{L} \right)^2 \frac{R}{p} \left( \Gamma_d - \frac{\partial T}{\partial p} \right) \quad (3.27)$$

and

$$B = \frac{8f_0^2}{(p_0 - p_T)^2}. \quad (3.28)$$

$$\frac{\omega_N}{\omega_d} = \frac{A_d + B}{B} \quad (3.33)$$

$$\sigma \equiv \frac{R}{p} \left( \left( \frac{dT}{dp} \right)_m - \frac{\partial T}{\partial p} \right) \quad (3.36)$$

$$\omega_m = \frac{forcing}{A_m + B} \quad (3.37)$$

$$A_m = 2 \left( \frac{2\pi}{L} \right)^2 \sigma = 2 \left( \frac{2\pi}{L} \right)^2 \frac{R}{p} \left( \Gamma_m - \frac{\partial T}{\partial p} \right) \quad (3.38)$$

$$\left(\frac{\partial \zeta_g}{\partial t}\right)_{p_0} = [-\mathbf{V}_g \cdot \nabla_p \eta_g]_{p_{ND}} + \frac{\nabla_p^2}{f_0} \int_{p_{ND}}^{p_0} \left( \frac{R}{P} (\mathbf{V}_g \cdot \nabla_p T) - \sigma \omega - \frac{\dot{Q}}{c_p \rho} \right) dp \quad (3.46)$$

$$\left(\frac{\partial \zeta_g}{\partial t}\right)_{p_0} = [-\mathbf{V}_g \cdot \nabla_p \eta_g]_{500} - \frac{g}{f} \nabla_p^2 \left( -\mathbf{V}_{g0} \cdot \nabla_p (Z_{500} - Z_{1000}) + \sigma \bar{\omega} + \frac{\dot{Q}}{c_p \rho} \right) \quad (3.47)$$

$$\frac{\partial(\zeta + f)}{\partial t} = -(\zeta + f) \nabla_p \cdot \mathbf{V}, \quad (3.49)$$

$$THRES = \frac{10 hPa}{6 h} \frac{\sin(\phi)}{\sin(60^\circ)}$$

$$\frac{d\theta}{dt} = \frac{\partial \theta}{\partial t} + \mathbf{V} \cdot \nabla_p \theta + \omega \frac{\partial \theta}{\partial p} = \left( \frac{\dot{Q}}{c_p} \right) \left( \frac{p_0}{p} \right)^\kappa. \quad (4.1)$$

$$\theta = T (p_0 / p)^{R/c_p}$$

$$\omega = \frac{\partial \theta / \partial t + \mathbf{V} \cdot \nabla_p \theta}{-\partial \theta / \partial p} \quad (4.2)$$

$$\omega_\theta = \left( \frac{\partial p}{\partial t} \right)_\theta + (\mathbf{V}_\theta \cdot \nabla_\theta p), \quad (4.3)$$

$$v_{g\theta} = \frac{1}{f_0} \frac{\partial M}{\partial x} \quad (4.4)$$

$$u_{g\theta} = -\frac{1}{f_0} \frac{\partial M}{\partial y} \quad (4.5)$$

$$M = (c_p T + g Z)_\theta \quad (4.6)$$

$$\frac{d}{dt} [(f + \zeta_\theta) \left( \frac{\partial \theta}{\partial p} \right)] = 0. \quad (4.7)$$

$$\frac{\delta S}{\delta t} = \frac{\partial S}{\partial t} + \mathbf{C} \cdot \nabla S \quad (4.8)$$

$$\left( \frac{\partial S}{\partial t} \right)_{\theta} = -\mathbf{C}_{\theta} \cdot \nabla_{\theta} S. \quad (4.9)$$

$$\left( \frac{\partial p}{\partial t} \right)_{\theta} = -C_{\theta x} \left( \frac{\partial p}{\partial x} \right)_{\theta}. \quad (4.10)$$

$$\omega_{\theta} = \mathbf{V}_{\mathbf{R}\theta} \cdot \nabla_{\theta} p \approx (\mathbf{V}_{\mathbf{R}} \cdot \nabla_p \theta) / \left( -\frac{\partial \theta}{\partial p} \right) \quad (4.11)$$

$$\mathbf{V}_{\mathbf{R}\theta} = \mathbf{V}_{\theta} - C_x \mathbf{i} - C_y \mathbf{j} \quad (4.12)$$

$$(A_E, A_Z) = R_d \int_m \frac{1}{\sigma} \left( [v_g^* T^*] \right) \frac{\partial [T]}{\partial y} dm \quad (1.30)$$

$$(A_E, K_E) = -R_d \int_m \frac{[\omega^* T^*]}{p} dm \quad (1.28)$$

$$\frac{\partial V_g}{\partial p} = \frac{R}{f p} \frac{\partial T}{\partial n} \quad (5.1)$$

$$V_{g0} - V_{gJ} = \int_{p_0}^{p_J} \frac{R}{f} \frac{\partial T}{\partial n} d \ln p = \frac{R}{f} \frac{\partial \bar{T}}{\partial n} \ln \left( \frac{p_0}{p_J} \right) \quad (5.2)$$

$$\frac{\partial T}{\partial t} = -\mathbf{V} \cdot \nabla_p T - \omega \frac{\partial T}{\partial p} + \frac{\omega}{\rho c_p} + \frac{\dot{Q}}{c_p}. \quad (5.3)$$

$$\frac{\partial V_{gJ}}{\partial n} \approx -\frac{R}{f} \ln \left( \frac{p_0}{p_J} \right) \frac{\partial^2 \bar{T}}{\partial n^2}. \quad (5.5)$$

$$\frac{dV}{dt} = -g \frac{\partial Z}{\partial s} + F_s \quad (5.6)$$

$$\frac{V^2}{R_T} = -g \frac{\partial Z}{\partial n} - fV + F_n. \quad (5.7)$$

$$\frac{\partial V_{gr}}{\partial p} = \frac{\frac{R}{f p} \frac{\partial T}{\partial n}}{\left(1 + \frac{2K_T V_{gr}}{f}\right)} \quad (5.9)$$

$$\tan \psi_i = \frac{\Delta z}{\Delta n}. \quad (5.10)$$

$$\frac{du}{dt} = f(v - v_g) = f v_{ag}$$

$$\tan \psi_i = -\frac{f \bar{T}}{g} \frac{\Delta_z V_g}{\Delta_n T} = \frac{f \bar{T}}{g} \left( \frac{V'_g - V_g}{(T' - T)_p} \right) \quad (5.12)$$

$$\tan \psi_i = \frac{\delta z}{\delta n} = -\frac{f \bar{T}}{g} \frac{\left. \frac{\partial V_g}{\partial z} \right|_{tr} - \left. \frac{\partial V_g}{\partial z} \right|_{str}}{\left. \frac{\partial T}{\partial z} \right|_{str} - \left. \frac{\partial T}{\partial z} \right|_{tr}} \quad (5.16) \quad \zeta = \frac{V}{R_s} - \frac{\partial V}{\partial n}$$

$$V_s \frac{\partial \eta}{\partial s} = -\eta \nabla \cdot \mathbf{V} \quad (5.18)$$

$$\begin{aligned} \frac{d}{dt} |\nabla_p \theta| &= \nabla_p \theta^{-1} \left\{ - \left[ \left( \frac{\partial \theta}{\partial x} \right)^2 \frac{\partial u}{\partial x} + \left( \frac{\partial \theta}{\partial y} \right)^2 \frac{\partial v}{\partial y} \right] - \left[ \frac{\partial \theta}{\partial x} \frac{\partial \theta}{\partial y} \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \right] \right. \\ &\quad - \left[ \left( \frac{\partial \theta}{\partial x} \right) \left( \frac{\partial \theta}{\partial p} \right) \frac{\partial \omega}{\partial x} + \left( \frac{\partial \theta}{\partial y} \right) \left( \frac{\partial \theta}{\partial p} \right) \frac{\partial \omega}{\partial y} \right] \\ &\quad \left. + \left[ \frac{\partial \theta}{\partial x} \frac{\partial}{\partial x} \left( \frac{d\theta}{dt} \right) + \frac{\partial \theta}{\partial y} \frac{\partial}{\partial y} \left( \frac{d\theta}{dt} \right) \right] \right\} \\ &= \nabla_p \theta^{-1} \{ +C + S + T + DB \}. \end{aligned} \quad (13.1)$$

$$\frac{d}{dt}\left(-\frac{\partial\theta}{\partial y}\right) = -\frac{\partial}{\partial y}\left(\frac{d\theta}{dt}\right) + \frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y} + \frac{\partial w}{\partial y}\frac{\partial\theta}{\partial z}. \quad (6.2)$$

$$u = \bar{u}\alpha y \quad (6.4)$$

$$v = 0 \quad (6.5)$$

$$\frac{d}{dt}\left(-\frac{\partial\theta}{\partial y}\right) = \bar{u}\alpha\frac{\partial\theta}{\partial x} \quad (6.6)$$

$$\left(-\frac{\partial\theta}{\partial y}\right)\bigg|_t = \left(-\frac{\partial\theta}{\partial y}\right)\bigg|_{t_0} + \bar{u}\alpha\frac{\partial\theta}{\partial x}[t-t_0] \quad (6.7)$$

$$u = \bar{v}\beta x \quad (6.8)$$

$$v = -\bar{v}\beta y \quad (6.9)$$

$$\frac{d}{dt}\left(-\frac{\partial\theta}{\partial y}\right) = -\bar{v}\beta\frac{\partial\theta}{\partial y}, \quad (6.10)$$

$$\left(\frac{\partial\theta}{\partial y}\right) = \left(\frac{\partial\theta}{\partial y}\right)_0 \exp^{\bar{v}\beta t} \quad (6.11)$$

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = -2\nabla_p \cdot \mathbf{Q} - \frac{R}{\sigma p}\beta\frac{\partial T}{\partial x}, \quad (6.14)$$

$$Q_i = -\frac{R}{\sigma p}\frac{\partial v_g}{\partial x}\frac{\partial T}{\partial y}, \quad (6.18)$$

$$Q_j = -\frac{R}{\sigma p}\frac{\partial v_g}{\partial y}\frac{\partial T}{\partial y}. \quad (6.19)$$

$$F = \frac{\sigma p}{R} \frac{1}{|\nabla_p T|} (\nabla_p T \cdot \mathbf{Q}), \quad (6.21)$$

$$F = \frac{\sigma p}{R} \frac{1}{|\nabla_p \theta|} (\nabla_p \theta \cdot \mathbf{Q}). \quad (6.23)$$

$$u_g = -\frac{1}{f} \frac{\partial \Phi}{\partial y} \quad (6.24)$$

$$\frac{du}{dt} - fv = \frac{d}{dt}(u - fy) = 0 \quad (6.25)$$

$$\frac{dv}{dt} - f(u_g - u) = 0 \quad (6.26)$$

$$\frac{dv}{dt} = -f\left(f - \frac{\partial u_g}{\partial y}\right)\Delta y. \quad (6.27)$$