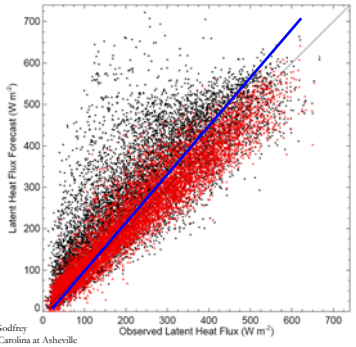
Linear Least Squares Example



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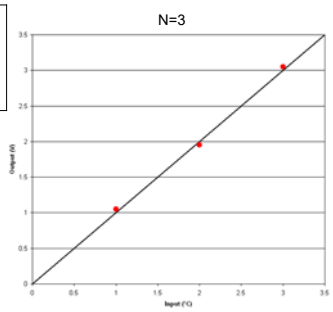
Linear Least Squares Example

Input X Temperature (°C)	Output Y Voltage (V)
1	1.05
2	1.95
3	3.05



The data don't quite fit the y = x line. This is a reasonable fit, but is this the best fit?

Calculate the intercept (a_0)

$$a_0 = \frac{\sum_{i=1}^{N} y_i \sum_{i=1}^{N} x_i^2 - \sum_{i=1}^{N} x_i \sum_{i=1}^{N} x_i y_i}{N\sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i\right)^2}$$

$$a_0 = \frac{(1.05+1.95+3.05)(1^2+2^2+3^2) - (1+2+3)(1*1.05+2*1.95+3*3.05)}{3(1^2+2^2+3^2) - (1+2+3)^2}$$

$$a_0 = \frac{84.7-84.6}{42-36} = 0.017$$

But wait…what are the units?

a_0 =

a_0 has the same units as the output (y-axis)

Calculate the slope (a_1)

$$a_1 = \frac{N\sum_{i=1}^{N} x_i y_i - \sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i}{N\sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i\right)^2}$$

$$a_1 = \frac{3[(1*1.05+2*1.95+3*3.05)] - [(1+2+3)(1.05+1.95+3.05)]}{3(1^2+2^2+3^2) - (1+2+3)^2}$$

$$a_1 = \frac{42.3-36.3}{42-36} = 1$$

Again, what are the units?

a_1 =

a_1 has the same units as y/x → It's the slope!

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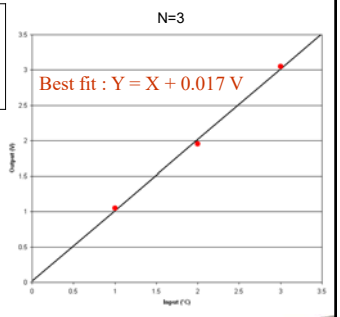

a_1 = Slope = 1 V °C^{-1}

a_0 = y - intercept = 0.017 V

In a calibration exercise, a_1 is the static sensitivity.