Formula Sheet 1 for Exam#3 **ATMS 411**

$$Z_t - Z_b = \frac{R\overline{T_v}}{g_0} ln\left(\frac{p_b}{p_t}\right), \tag{3.1}$$

$$p_h = \int_h^\infty g\rho dz. \tag{3.2}$$

$$\frac{\partial p_s}{\partial t} \approx -\int_0^{p_s} \nabla_p \cdot \mathbf{V}\ dp, \tag{3.3}$$

$$\frac{\delta p_s}{\delta t} = \frac{\partial p_s}{\partial t} + \mathbf{C} \cdot \nabla_z p_s \tag{3.4}$$

$$\frac{\partial p_s}{\partial t} = -\mathbf{C} \cdot \nabla_z p_s. \tag{3.5}$$

$$\begin{aligned} \left(\nabla_p^2 + \frac{{f_0}^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega &= \frac{f_0}{\sigma}\frac{\partial}{\partial p}\left(\mathbf{V}_g \cdot \nabla_p \left(\frac{\nabla_p^2\Phi}{f_0} + f\right)\right) \\ &+ \frac{1}{\sigma}\nabla_p^2\left(\mathbf{V}_g \cdot \nabla_p \left(-\frac{\partial\Phi}{\partial p}\right)\right) \\ &- \frac{1}{\sigma}\nabla_p^2\left(\frac{\dot{Q}}{\rho c_p}\right) \\ &- \frac{f_0}{\sigma}\frac{\partial}{\partial p}(\mathbf{k} \cdot \nabla_p \times \mathbf{F}). \end{aligned} \tag{3.6}$$

$$\omega(x,y,p) = \omega_0 \sin\left(\frac{2\pi x}{L}\right)\sin\left(\frac{2\pi y}{L}\right)\left(\frac{(p_0 - p)(p - p_t)}{(\frac{1}{2})^2(p_0 - p_t)^2}\right) \tag{3.7}$$

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = -\left(\omega_v + \omega_T + \omega_H + \omega_f\right) \tag{3.8}$$

$$\omega_v = \frac{f_0}{\sigma}\frac{\partial}{\partial p}\left(-\mathbf{V}_g \cdot \nabla_p \left(\frac{\nabla_p^2\Phi}{f_0} + f\right)\right) = \frac{f_0}{\sigma}\frac{\partial}{\partial p}(-\mathbf{V}_g \cdot \nabla_p \eta_g). \tag{3.13}$$

$$\omega_T = \frac{-1}{\sigma}\nabla_p^2\left(\mathbf{V}_g \cdot \nabla_p \left(-\frac{\partial\Phi}{\partial p}\right)\right) = \frac{R}{\sigma p}\nabla_p^2(-\mathbf{V}_g \cdot \nabla_p T) \tag{3.17}$$

$$\left(\nabla_p^2 + \frac{{f_0}^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = \frac{2}{\sigma}\frac{\partial \mathbf{V}_g}{\partial p} \cdot \nabla_p \eta_g. \tag{3.19}$$

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\frac{\partial \Phi}{\partial t} = -f_0 \mathbf{V}_g \cdot \nabla_p \eta_g + \frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left(\mathbf{V}_g \cdot \nabla_p \left(-\frac{\partial \Phi}{\partial p}\right)\right) - \frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left(\frac{\dot{Q}}{\rho c_p}\right) \quad (3.20)$$

$$\sigma = \frac{1}{\theta}\frac{\partial \Phi}{\partial p}\frac{\partial \theta}{\partial p} = g\frac{\partial z}{\partial p}\left(\frac{1}{T}\frac{\partial T}{\partial p} - \frac{\kappa}{p}\frac{\partial p}{\partial p}\right) = \frac{R}{p}\left(\Gamma_d - \frac{\partial T}{\partial p}\right) \quad (3.21)$$

$$\left(-2\left(\frac{2\pi}{L}\right)^2 \sigma + \left(\frac{-8 f_0^2}{(p_0 - p_T)^2}\right)\right)\omega_d = -forcing = -\sigma(\omega_v + \omega_T + \omega_H + \omega_f) \quad (3.25)$$

which can be written in shorthand as

$$(A_d + B)\omega_d = +forcing \quad (3.26)$$

$$A_d = 2\left(\frac{2\pi}{L}\right)^2 \sigma = 2\left(\frac{2\pi}{L}\right)^2 \frac{R}{p}\left(\Gamma_d - \frac{\partial T}{\partial p}\right) \quad (3.27)$$

and

$$B = \frac{8 f_0^2}{(p_0 - p_T)^2}. \quad (3.28)$$

$$\frac{\omega_N}{\omega_d} = \frac{A_d + B}{B} \quad (3.33)$$

$$\sigma \equiv \frac{R}{p}\left(\left(\frac{dT}{dp}\right)_m - \frac{\partial T}{\partial p}\right) \quad (3.36)$$

$$\omega_m = \frac{forcing}{A_m + B} \quad (3.37)$$

$$A_m = 2\left(\frac{2\pi}{L}\right)^2 \sigma = 2\left(\frac{2\pi}{L}\right)^2 \frac{R}{p}\left(\Gamma_m - \frac{\partial T}{\partial p}\right) \quad (3.38)$$

$$\left(\frac{\partial \zeta_g}{\partial t}\right)_{p_0} = [-\mathbf{V}_g \cdot \nabla_p \eta_g]_{p_{ND}} + \frac{\nabla_p^2}{f_0}\int_{p_{ND}}^{p_0}\left(\frac{R}{P}(\mathbf{V}_g \cdot \nabla_p T) - \sigma\omega - \frac{\dot{Q}}{c_p \rho}\right) dp \quad (3.46)$$

$$\left(\frac{\partial \zeta_g}{\partial t}\right)_{p_0} = [-\mathbf{V}_g \cdot \nabla_p \eta_g]_{500} - \frac{g}{f}\nabla_p^2\left(-\mathbf{V}_{g_0} \cdot \nabla_p (Z_{500} - Z_{1000}) + \bar{\sigma}\bar{\omega} + \frac{\bar{\dot{Q}}}{c_p \rho}\right) \quad (3.47)$$

$$\frac{\partial(\zeta + f)}{\partial t} = -(\zeta + f)\nabla_{\mathbf{p}} \cdot \mathbf{V}, \quad (3.49)$$

$$THRES = \frac{10\,hPa}{6\,h}\frac{\sin(\phi)}{\sin(60^o)}$$

$$\frac{d\theta}{dt} = \frac{\partial \theta}{\partial t} + \mathbf{V} \cdot \nabla_p \theta + \omega\frac{\partial \theta}{\partial p} = \left(\frac{\dot{Q}}{c_p}\right)\left(\frac{p_0}{p}\right)^{\kappa}. \quad (4.1)$$

$$\theta = T\,(p_0 / p)^{R/cp}$$

$$\omega = \frac{\partial\theta/\partial t + \mathbf{V} \cdot \nabla_p \theta}{-\partial\theta/\partial p} \quad (4.2)$$

$$\omega_\theta = \left(\frac{\partial p}{\partial t}\right)_\theta + (\mathbf{V}_\theta \cdot \nabla_\theta p), \quad (4.3)$$

$$v_{g\theta} = \frac{1}{f_0}\frac{\partial M}{\partial x} \quad (4.4)$$

$$u_{g\theta} = -\frac{1}{f_0}\frac{\partial M}{\partial y} \quad (4.5)$$

$$M = (c_p\,T + g\,Z)_\theta \quad (4.6)$$

$$\frac{d}{dt}[(f + \zeta_\theta)(\frac{\partial \theta}{\partial p})] = 0. \quad (4.7)$$

$$\frac{\delta S}{\delta t} = \frac{\partial S}{\partial t} + \mathbf{C} \cdot \nabla S \tag{4.8}$$

$$\left(\frac{\partial S}{\partial t}\right)_\theta = -\mathbf{C}_\theta \cdot \nabla_\theta S. \tag{4.9}$$

$$\left(\frac{\partial p}{\partial t}\right)_\theta = -C_{\theta x}\left(\frac{\partial p}{\partial x}\right)_\theta . \tag{4.10}$$

$$\omega_\theta = \mathbf{V}_{\mathbf{R}\theta} \cdot \nabla_\theta p \approx (\mathbf{V}_\mathbf{R} \cdot \nabla_p \theta)/(-\frac{\partial \theta}{\partial p}) \tag{4.11}$$

$$\mathbf{V}_{\mathbf{R}\theta} = \mathbf{V}_\theta - C_x \mathbf{i} - C_y \mathbf{j} \tag{4.12}$$

$$(A_E, A_Z) = R_d \int_m \frac{1}{\sigma}\left([v_g^* T^*]\right)\frac{\partial [T]}{\partial y} dm \tag{1.30}$$

$$(A_E, K_E) = -R_d \int_m \frac{[\omega^* T^*]}{p} dm \tag{1.28}$$

$$\frac{\partial V_g}{\partial p} = \frac{R}{fp}\frac{\partial T}{\partial n} \tag{5.1}$$

$$V_{g0} - V_{gJ} = \int_{p_0}^{p_J} \frac{R}{f}\frac{\partial T}{\partial n} d\ln p = \frac{R}{f}\frac{\partial \overline{T}}{\partial n}\ln(\frac{p_0}{p_J}) \tag{5.2}$$

$$\frac{\partial T}{\partial t} = -\mathbf{V} \cdot \nabla_p T - \omega \frac{\partial T}{\partial p} + \frac{\omega}{\rho c_p} + \frac{\dot{Q}}{c_p}. \tag{5.3}$$

$$\frac{\partial V_{gJ}}{\partial n} \approx -\frac{R}{f}\ln(\frac{p_0}{p_J})\frac{\partial^2 \overline{T}}{\partial n^2}. \tag{5.5}$$

$$\frac{dV}{dt} = -g\frac{\partial Z}{\partial s} + F_s \qquad (5.6)$$

$$\frac{V^2}{R_T} = -g\frac{\partial Z}{\partial n} - fV + F_n. \qquad (5.7)$$

$$\frac{\partial V_{gr}}{\partial p} = \frac{\frac{R}{fp}\frac{\partial T}{\partial n}}{\left(1 + \frac{2K_T V_{gr}}{f}\right)} \qquad (5.9)$$

$$\tan\psi_i = \frac{\Delta z}{\Delta n}. \qquad (5.10)$$

$$\frac{du}{dt} = f\left(v - v_g\right) = f\,v_{ag}$$

$$\tan\psi_i = -\frac{f\overline{T}}{g}\frac{\Delta_z V_g}{\Delta_n T} = \frac{f\overline{T}}{g}\left(\frac{V_g' - V_g}{(T' - T)_p}\right) \qquad (5.12)$$

$$\tan\psi_i = \frac{\delta z}{\delta n} = -\frac{f\overline{T}}{g}\frac{\left.\frac{\partial V_g}{\partial z}\right|_{tr} - \left.\frac{\partial V_g}{\partial z}\right|_{str}}{\left.\frac{\partial T}{\partial z}\right|_{str} - \left.\frac{\partial T}{\partial z}\right|_{tr}} \qquad (5.16)$$

$$\zeta = \frac{V}{R_s} - \frac{\partial V}{\partial n}$$

$$V_s\frac{\partial \eta}{\partial s} = -\eta\nabla\cdot\mathbf{V} \qquad (5.18)$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}|\nabla_p\theta| = \nabla_p\theta^{-1}\Bigg\{ &-\left[\left(\frac{\partial\theta}{\partial x}\right)^2\frac{\partial u}{\partial x} + \left(\frac{\partial\theta}{\partial y}\right)^2\frac{\partial v}{\partial y}\right] - \left[\frac{\partial\theta}{\partial x}\frac{\partial\theta}{\partial y}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right] \\ &- \left[\left(\frac{\partial\theta}{\partial x}\right)\left(\frac{\partial\theta}{\partial p}\right)\frac{\partial\omega}{\partial x} + \left(\frac{\partial\theta}{\partial y}\right)\left(\frac{\partial\theta}{\partial p}\right)\frac{\partial\omega}{\partial y}\right] \\ &+ \left[\frac{\partial\theta}{\partial x}\frac{\partial}{\partial x}\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right) + \frac{\partial\theta}{\partial y}\frac{\partial}{\partial y}\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)\right]\Bigg\} \qquad (13.1) \\ = \nabla_p\theta^{-1}\{ &+ C + S + T + DB\}.\end{aligned}$$

$$\frac{d}{dt}(-\frac{\partial\theta}{\partial y}) = -\frac{\partial}{\partial y}(\frac{d\theta}{dt}) + \frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y} + \frac{\partial w}{\partial y}\frac{\partial\theta}{\partial z}. \qquad (6.2)$$

$$u = \overline{u}\alpha y \qquad (6.4)$$

$$v = 0 \qquad (6.5)$$

$$\frac{d}{dt}(-\frac{\partial\theta}{\partial y}) = \overline{u}\alpha\frac{\partial\theta}{\partial x} \qquad (6.6)$$

$$\left(-\frac{\partial\theta}{\partial y}\right)\Bigg|_{t} = \left(-\frac{\partial\theta}{\partial y}\right)\Bigg|_{t0} + \overline{u}\,\alpha\,\frac{\partial\theta}{\partial x}\left[t - t0\right] \qquad (6.7)$$

$$u = \overline{v}\beta x \qquad (6.8)$$

$$v = -\overline{v}\beta y \qquad (6.9)$$

$$\frac{d}{dt}(-\frac{\partial\theta}{\partial y}) = -\overline{v}\beta\frac{\partial\theta}{\partial y}, \qquad (6.10)$$

$$\left(\frac{\partial\theta}{\partial y}\right) = \left(\frac{\partial\theta}{\partial y}\right)_0 \exp^{\overline{v}\beta t} \qquad (6.11)$$

$$\left(\nabla_p^2 + \frac{f_0{}^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = -2\nabla_p \cdot \mathbf{Q} - \frac{R}{\sigma p}\beta\frac{\partial T}{\partial x}, \qquad (6.14)$$

$$Q_i = -\frac{R}{\sigma p}\frac{\partial v_g}{\partial x}\frac{\partial T}{\partial y}, \qquad (6.18)$$

$$Q_j = -\frac{R}{\sigma p}\frac{\partial v_g}{\partial y}\frac{\partial T}{\partial y}. \qquad (6.19)$$

$$F = \frac{\sigma p}{R} \frac{1}{|\nabla_p T|} (\nabla_p T \cdot \mathbf{Q}), \qquad (6.21)$$

$$F = \frac{\sigma p}{R} \frac{1}{|\nabla_p T|} (\nabla_p T \cdot \mathbf{Q}), \qquad (6.21)$$

$$F = \frac{\sigma p}{R} \frac{1}{|\nabla_p \theta|} (\nabla_p \theta \cdot \mathbf{Q}). \qquad (6.23)$$

$$u_g = -\frac{1}{f} \frac{\partial \Phi}{\partial y} \qquad (6.24)$$

$$\frac{du}{dt} - fv = \frac{d}{dt}(u - fy) = 0 \qquad (6.25)$$

$$\frac{dv}{dt} - f(u_g - u) = 0 \qquad (6.26)$$

$$\frac{dv}{dt} = -f(f - \frac{\partial u_g}{\partial y})\Delta y. \qquad (6.27)$$

$$\frac{\overline{D}\overline{u}}{Dt} = -\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial x} + f\overline{v} - \frac{\partial \overline{u'w'}}{\partial z} \qquad \frac{\overline{D}\overline{v}}{Dt} = -\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial y} - f\overline{u} - \frac{\partial \overline{v'w'}}{\partial z}$$

$$f\left(\overline{v} - \overline{v}_g\right) - \frac{\partial \overline{u'w'}}{\partial z} = 0 \qquad -f\left(\overline{u} - \overline{u}_g\right) - \frac{\partial \overline{v'w'}}{\partial z} = 0$$

$$\frac{\overline{D}\,\overline{\theta}}{Dt} = -\overline{w}\frac{d\theta_0}{dz} - \frac{LE}{\rho C_p} - \frac{\partial \overline{w'\theta'}}{\partial z} \qquad \frac{\overline{D}\overline{q}}{Dt} = -\overline{w}\frac{dq_0}{dz} + \frac{E}{\rho} - \frac{\partial \overline{w'q'}}{\partial z}$$

$$\frac{\overline{D}(TKE)}{Dt} = MP + BPL + TR - \varepsilon$$

$$BPL = \overline{w'\theta'}\left(\frac{g}{\theta_0}\right), \qquad MP = -\overline{u'w'}\frac{\partial \overline{u}}{\partial z} - \overline{v'w'}\frac{\partial \overline{v}}{\partial z}$$

$$f\left(\bar{v}-\bar{v}_g\right) = \frac{C_d\left|\bar{V}\right|\bar{u}}{h}$$

$$-f\left(\bar{u}-\bar{u}_g\right)= \frac{C_d\left|\bar{V}\right|\bar{v}}{h}$$

$$\bar{v} = \frac{C_d\left|\bar{V}\right|\bar{u}}{f\,h}$$

$$\bar{u} = \bar{u}_g - \frac{C_d\left|\bar{V}\right|\bar{v}}{f\,h}$$

$$K_m \frac{\partial^2\bar{u}}{\partial z^2} + f\left(\bar{v}-\bar{v}_g\right)=0$$

$$K_m \frac{\partial^2\bar{v}}{\partial z^2} - f\left(\bar{u}-\bar{u}_g\right)=0$$

$$\bar{v} = \bar{u}_g e^{-\gamma z}\sin\gamma z$$

$$\bar{u} = \bar{u}_g\left(1-e^{-\gamma z}\cos\gamma z\right)$$

$$K_m = \frac{\partial\bar{u}}{\partial z}$$

$$u_*^2 \equiv \left|\overline{\left(u'w'\right)_s}\right| = K_m \frac{\partial\bar{u}}{\partial z}$$

$$\frac{\partial\bar{u}}{\partial z} = \frac{u_*}{kz} \;\Rightarrow \text{after integrating w.r.t. } z \Rightarrow \bar{u} = \frac{u_*}{k}\ln\left(\frac{z}{z_0}\right)$$

$$w(De) = \zeta_g \left|\frac{K_m}{2f}\right|^{1/2}\left(\frac{f}{|f|}\right)$$

$$\zeta_g(t) = \zeta_g(0)\exp(-t/\tau)$$

$$\frac{d(\zeta+f)}{dt} = -(\zeta+f)\nabla\bullet\vec{\mathbf{V}} + \hat{\mathbf{k}}\bullet\left(\nabla\times\overrightarrow{\mathbf{Fr}}\right)$$

$$v_g = \frac{g}{f}\left(\frac{\partial Z}{\partial x}\right)_p \tag{2.6}$$

$$u_g = -\frac{g}{f}\left(\frac{\partial Z}{\partial y}\right)_p \tag{2.7}$$