Formula Sheet 1 for Exam#3 **ATMS 410**

$$[x] = \frac{1}{2\pi a \cos\phi} \int_0^{2\pi} x a \cos\phi d\lambda \qquad (1.1)$$

$$\overline{x} = \frac{1}{T} \int_0^{T} x dt \qquad (1.2)$$

$$[inflow + sources] - [outflow + sinks] - [storage] = 0 \qquad (1.3)$$

$$M = a\cos\phi(\Omega a \cos\phi + u) \qquad (1.4)$$

$$u_f = \frac{\Omega a^2 (\cos^2\phi_i - \cos^2\phi_f) + u_i(a\cos\phi_i)}{(a\cos\phi_f)} \qquad (1.5)$$

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} - \omega\frac{\partial T}{\partial p} + \gamma_d\omega + \frac{\dot{Q}}{c_p} \qquad (1.6)$$

$$\frac{\partial T}{\partial t} = -\frac{\partial uT}{\partial x} - \frac{\partial vT}{\partial y} - \frac{\partial \omega T}{\partial p} + \gamma_d\omega + \frac{\dot{Q}}{c_p}. \qquad (1.7)$$

$$uT = ([u] + u^*)([T] + T^*) = [u][T] + u^*[T] + [u]T^* + u^*T^* \qquad (1.13)$$

$$[uT] = [[u][T]] + [u^*[T]] + [[u]T^*] + [u^*T^*] = [u][T] + [u^*T^*]. \qquad (1.14)$$

$$\frac{\partial [T]}{\partial t} = -\frac{\partial [v][T]}{\partial y} - \frac{\partial [\omega][T]}{\partial p} + \gamma_d[\omega] + [\frac{\dot{Q}}{c_p}] - \frac{\partial [v^*T^*]}{\partial y} - \frac{\partial [\omega^*T^*]}{\partial p} \qquad (1.15)$$

$$\frac{\partial [u]}{\partial t} = -\frac{1}{\cos^2\phi}\frac{\partial [u][v]\cos^2\phi}{\partial y} - \frac{\partial [u][\omega]}{\partial p} + f[v] + [F_x] - \frac{1}{\cos^2\phi}\frac{\partial [u^*v^*]\cos^2\phi}{\partial y} - \frac{\partial [u^*\omega^*]}{\partial p} \qquad (1.16)$$

$$\overline{uT} = \overline{(\overline{u} + u')(\overline{T} + T')} = \overline{\overline{u}\overline{T}} + \overline{u'\overline{T}} + \overline{\overline{u}T'} + \overline{u'T'} \qquad (1.17)$$

$$\overline{uT} = \overline{(\overline{u} + u')(\overline{T} + T')} = \overline{u}\overline{T} + \overline{u'T'} \qquad (1.18)$$

$$\overline{[v][T]} = \overline{[v]}\ \overline{[T]} + \overline{[v]'[T]'} \tag{1.21}$$

and

$$\overline{[v^*T^*]} = [\overline{v^*}\ \overline{T^*}] + [\overline{v^{*\prime}T^{*\prime}}] \tag{1.22}$$

$$\begin{aligned} 0 = \frac{\partial\overline{[T]}}{\partial t} = & -\frac{\partial\overline{[v]}\ \overline{[T]}}{\partial y} - \frac{\partial\overline{[\omega]}\ \overline{[T]}}{\partial p} + \gamma_d\overline{[\omega]} + \frac{\overline{[\dot{Q}]}}{c_p} \\ & -\frac{\partial\overline{[v]'[T]'}}{\partial y} - \frac{\partial\overline{[\omega]'[T]'}}{\partial p} \\ & -\frac{\partial[\overline{v^*}\overline{T^*}]}{\partial y} - \frac{\partial[\overline{\omega^*}\overline{T^*}]}{\partial p} \\ & -\frac{\partial[\overline{v^{*\prime}T^{*\prime}}]}{\partial y} - \frac{\partial[\overline{\omega^{*\prime}T^{*\prime}}]}{\partial p} \end{aligned} \tag{1.23}$$

$$\begin{aligned} 0 = \frac{\partial\overline{[u]}}{\partial t} = & -\frac{1}{\cos^2\phi}\frac{\partial\overline{[u]}\ \overline{[v]}\cos^2\phi}{\partial y} - \frac{\partial\overline{[u]}\ \overline{[\omega]}}{\partial p} + f\overline{[v]} + \overline{[F_x]} \\ & -\frac{1}{\cos^2\phi}\frac{\partial\overline{[u]'[v]'}\cos^2\phi}{\partial y} - \frac{\partial\overline{[u]'[\omega]'}}{\partial p} \\ & -\frac{1}{\cos^2\phi}\frac{\partial[\overline{u^*}\ \overline{v^*}]\cos^2\phi}{\partial y} - \frac{\partial[\overline{u^*}\ \overline{\omega^*}]}{\partial p} \\ & -\frac{1}{\cos^2\phi}\frac{\partial[\overline{u^{*\prime}v^{*\prime}}]\cos^2\phi}{\partial y} - \frac{\partial[\overline{u^{*\prime}\omega^{*\prime}}]}{\partial p} \end{aligned} \tag{1.24}$$

Formula Sheet 3 for Exam#3 **ATMS 410**

$$\text{NAO index} = \text{SLP}'_{AZ} - \text{SLP}'_{ICE}$$

$$\text{PNA index} = \frac{Z'_{(1)} - Z'_{(2)} + Z'_{(3)} - Z'_{(4)}}{4}$$

$$\bar{P} \propto \frac{1}{V} \int \frac{\overline{\theta'^2}}{\bar{\theta}^2}\, dV$$

$$E_\text{I} = c_v \int_0^\infty \rho T\, dz$$

$$E_\text{p} = \int_0^\infty p\, dz = R \int_0^\infty \rho T\, dz$$

$$E_\text{p} + E_\text{I} = \frac{c_p}{c_v} E_\text{I} = \frac{c_p}{R} E_\text{p}$$

$$(K_E, K_Z) = \int_m [u_g^* v_g^*] \frac{\partial [u_g]}{\partial y} dm \tag{1.25}$$

$$(A_Z, K_Z) = -R_d \int_m \frac{[\omega][T]}{p} dm \tag{1.26}$$

$$(D_Z) = \int_m [u_g][F_x] dm \tag{1.27}$$

$$(A_E, K_E) = -R_d \int_m \frac{[\omega^* T^*]}{p} dm \tag{1.28}$$

$$(D_E) = \int_m \left([\mathbf{V}_g^* \cdot \mathbf{F}^*] \right) dm \tag{1.29}$$

$$(A_E, A_Z) = R_d \int_m \frac{1}{\sigma} \left([v_g^* T^*] \right) \frac{\partial [T]}{\partial y} dm \tag{1.30}$$

$$(G_Z) = \frac{R_d}{c_p} \int_m \frac{[\dot{Q}_{db}][T]}{\sigma p} dm \tag{1.31}$$

$$(G_E) = \frac{R_d}{c_p} \int_m \frac{[\dot{Q}_{db}^* T^*]}{\sigma p} dm \tag{1.32}$$

$$\frac{du}{dt} = fv - \frac{1}{\rho}\frac{\partial p}{\partial x} + F_x \quad (2.1)$$

$$\frac{dv}{dt} = -fu - \frac{1}{\rho}\frac{\partial p}{\partial y} + F_y \quad (2.2)$$

$$\frac{dw}{dt} = -fu\cot\phi - \frac{1}{\rho}\frac{\partial p}{\partial z} - g + F_z \quad (2.3)$$

$$v_g = \frac{1}{f\rho}\frac{\partial p}{\partial x} \quad (2.4)$$

$$u_g = -\frac{1}{f\rho}\frac{\partial p}{\partial y} \quad (2.5)$$

$$v_g = \frac{g}{f}\left(\frac{\partial Z}{\partial x}\right)_p \quad (2.6)$$

$$u_g = -\frac{g}{f}\left(\frac{\partial Z}{\partial y}\right)_p \quad (2.7)$$

$$\nabla_p \cdot \mathbf{V} = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = -\frac{\partial \omega}{\partial p} = D_p \quad (2.8)$$

$$D_{pg} = -(\beta/f)v_g \quad (2.9)$$

$$\mathbf{V} = \mathbf{V}_\psi + \mathbf{V}_\chi \quad (2.11)$$

$$\nabla \cdot \mathrm{V}_\psi = 0 \qquad \nabla \times \mathrm{V}_\chi = 0$$

$$\mathbf{V}_\psi = \mathbf{k}\times\nabla\psi \quad (2.12)$$

$$u_\psi = -\frac{\partial \psi}{\partial y}, \quad v_\psi = \frac{\partial \psi}{\partial x} \quad (2.13)$$

$$\zeta = \mathbf{k}\cdot\nabla\times\mathbf{V} = \nabla^2\psi. \qquad (2.14)$$

$$\nabla^2\chi = -\nabla\cdot\mathbf{V} \qquad (2.15)$$

$$u_\chi = -\frac{\partial\chi}{\partial x}, \quad v_\chi = -\frac{\partial\chi}{\partial y} \qquad (2.16)$$

$$\frac{\partial p}{\partial z} = -\rho g \qquad (2.17)$$

$$\frac{du}{dt} = fv - g\frac{\partial Z}{\partial x} + F_x = f\left(v - v_g\right) + F_x = f v_{ag} + F_x$$

$$\frac{dv}{dt} = -fu - g\frac{\partial Z}{\partial y} + F_y = -f\left(u - u_g\right) + F_y = -f u_{ag} + F_y$$

$$\mathbf{V_T} = \frac{\partial \mathbf{V_g}}{\partial \ln p} = -\hat{k}\times\frac{R}{f}\frac{\partial \overline{T}_v}{\partial n} \qquad (2.18)$$

$$\frac{\partial(\zeta+f)}{\partial t} = -\mathbf{V}\cdot\nabla_p(\zeta+f) - \omega\frac{\partial\zeta}{\partial p} - (\zeta+f)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) - \left(\frac{\partial\omega}{\partial x}\frac{\partial v}{\partial p} - \frac{\partial u}{\partial p}\frac{\partial\omega}{\partial y}\right) + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right). \qquad (2.19)$$

$$\frac{\partial\zeta_g}{\partial t} = -\mathbf{V}_g\cdot\nabla_p\eta_g - f_0\nabla_p\cdot\mathbf{V} + f_0\mathbf{k}\cdot(\nabla_p\times\mathbf{F}) \qquad (2.20)$$

$\zeta_g = \dfrac{\nabla_p^2\Phi}{f_0}$, where $Z \equiv \dfrac{\Phi(z)}{g_0}$, $g_0 = 9.81\,\mathrm{m\,s^{-2}}$ $\qquad \eta_g = \zeta_g + f = \dfrac{\nabla_p^2\Phi}{f_0} + f$

$$\frac{\partial}{\partial t}\left(-\frac{\partial\Phi}{\partial p}\right) = -\mathbf{V}_g\cdot\nabla_p\left(-\frac{\partial\Phi}{\partial p}\right) + \sigma\omega + \frac{\dot{Q}}{c_p\rho} \qquad (2.21)$$

$$\frac{\partial T}{\partial t} = -\mathbf{V}_g\cdot\nabla_p T + \frac{\omega}{c_p\rho} + \frac{\dot{Q}}{c_p} \qquad (2.22)$$

Formula Sheet 6 for Exam#3 **ATMS 410**

$$Z_t - Z_b = \frac{R\overline{T_v}}{g_0} ln\left(\frac{p_b}{p_t}\right), \quad (3.1)$$

$$p_h = \int_h^\infty g\rho dz. \quad (3.2)$$

$$\frac{\partial p_s}{\partial t} \approx -\int_0^{p_s} \nabla_p \cdot \mathbf{V}\; dp, \quad (3.3)$$

$$\frac{\delta p_s}{\delta t} = \frac{\partial p_s}{\partial t} + \mathbf{C} \cdot \nabla_z p_s \quad (3.4)$$

$$\frac{\partial p_s}{\partial t} = -\mathbf{C} \cdot \nabla_z p_s. \quad (3.5)$$

$$\begin{aligned}\left(\nabla_p^2 + \frac{{f_0}^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega &= \frac{f_0}{\sigma}\frac{\partial}{\partial p}\left(\mathbf{V}_g \cdot \nabla_p\left(\frac{\nabla_p^2\Phi}{f_0} + f\right)\right) \\ &+ \frac{1}{\sigma}\nabla_p^2\left(\mathbf{V}_g \cdot \nabla_p\left(-\frac{\partial\Phi}{\partial p}\right)\right) \\ &- \frac{1}{\sigma}\nabla_p^2\left(\frac{\dot{Q}}{\rho c_p}\right) \\ &- \frac{f_0}{\sigma}\frac{\partial}{\partial p}(\mathbf{k} \cdot \nabla_p \times \mathbf{F}). \quad (3.6)\end{aligned}$$

$$\omega(x,y,p) = \omega_0 \sin\left(\frac{2\pi x}{L}\right)\sin\left(\frac{2\pi y}{L}\right)\left(\frac{(p_0 - p)(p - p_t)}{(\frac{1}{2})^2(p_0 - p_t)^2}\right) \quad (3.7)$$

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = -\left(\omega_v + \omega_T + \omega_H + \omega_f\right) \quad (3.8)$$

$$\omega_v = \frac{f_0}{\sigma}\frac{\partial}{\partial p}\left(-\mathbf{V}_g \cdot \nabla_p\left(\frac{\nabla_p^2\Phi}{f_0} + f\right)\right) = \frac{f_0}{\sigma}\frac{\partial}{\partial p}(-\mathbf{V}_g \cdot \nabla_p \eta_g). \quad (3.13)$$

$$\omega_T = \frac{-1}{\sigma}\nabla_p^2\left(\mathbf{V}_g \cdot \nabla_p\left(-\frac{\partial\Phi}{\partial p}\right)\right) = \frac{R}{\sigma p}\nabla_p^2(-\mathbf{V}_g \cdot \nabla_p T) \quad (3.17)$$

$$\left(\nabla_p^2 + \frac{{f_0}^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = \frac{2}{\sigma}\frac{\partial \mathbf{V}_g}{\partial p} \cdot \nabla_p \eta_g. \quad (3.19)$$

Formula Sheet 7 for Exam#3 **ATMS 410**

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\frac{\partial \Phi}{\partial t} = -f_0 \mathbf{V}_g \cdot \nabla_p \eta_g + \frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left(\mathbf{V}_g \cdot \nabla_p \left(-\frac{\partial \Phi}{\partial p}\right)\right)$$

$$-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left(\frac{\dot{Q}}{\rho c_p}\right) \qquad (3.20)$$

$$\sigma = \frac{1}{\theta}\frac{\partial \Phi}{\partial p}\frac{\partial \theta}{\partial p} = g\frac{\partial z}{\partial p}\left(\frac{1}{T}\frac{\partial T}{\partial p} - \frac{\kappa}{p}\frac{\partial p}{\partial p}\right) = \frac{R}{p}\left(\Gamma_d - \frac{\partial T}{\partial p}\right) \qquad (3.21)$$

$$\left(-2\left(\frac{2\pi}{L}\right)^2 \sigma + \left(\frac{-8f_0^2}{(p_0 - p_T)^2}\right)\right)\omega_d = -forcing = -\sigma(\omega_v + \omega_T + \omega_H + \omega_f) \qquad (3.25)$$

which can be written in shorthand as

$$(A_d + B)\omega_d = +forcing \qquad (3.26)$$

$$A_d = 2\left(\frac{2\pi}{L}\right)^2 \sigma = 2\left(\frac{2\pi}{L}\right)^2 \frac{R}{p}\left(\Gamma_d - \frac{\partial T}{\partial p}\right) \qquad (3.27)$$

and

$$B = \frac{8f_0^2}{(p_0 - p_T)^2}. \qquad (3.28)$$

$$\frac{\omega_N}{\omega_d} = \frac{A_d + B}{B} \qquad (3.33)$$

$$\sigma \equiv \frac{R}{p}\left(\left(\frac{dT}{dp}\right)_m - \frac{\partial T}{\partial p}\right) \qquad (3.36)$$

$$\omega_m = \frac{forcing}{A_m + B} \qquad (3.37)$$

$$A_m = 2\left(\frac{2\pi}{L}\right)^2 \sigma = 2\left(\frac{2\pi}{L}\right)^2 \frac{R}{p}\left(\Gamma_m - \frac{\partial T}{\partial p}\right) \qquad (3.38)$$

$$\left(\frac{\partial \zeta_g}{\partial t}\right)_{p_0} = [-\mathbf{V}_g \cdot \nabla_p \eta_g]_{p_{ND}} + \frac{\nabla_p^2}{f_0}\int_{p_{ND}}^{p_0}\left(\frac{R}{P}(\mathbf{V}_g \cdot \nabla_p T) - \sigma\omega - \frac{\dot{Q}}{c_p \rho}\right) dp \quad (3.46)$$

$$\left(\frac{\partial \zeta_g}{\partial t}\right)_{p_0} = [-\mathbf{V}_g \cdot \nabla_p \eta_g]_{500} - \frac{g}{f}\nabla_p^2\left(-\mathbf{V}_{g_0} \cdot \nabla_p (Z_{500} - Z_{1000}) + \bar{\sigma}\bar{\omega} + \frac{\bar{\dot{Q}}}{c_p \rho}\right) \quad (3.47)$$

$$\frac{\partial(\zeta + f)}{\partial t} = -(\zeta + f)\nabla_{\mathbf{p}} \cdot \mathbf{V}, \quad (3.49)$$

$$THRES = \frac{10\,hPa}{6\,h}\frac{\sin(\phi)}{\sin(60^o)}$$

$$\frac{d\theta}{dt} = \frac{\partial\theta}{\partial t} + \mathbf{V} \cdot \nabla_p \theta + \omega\frac{\partial\theta}{\partial p} = \left(\frac{\dot{Q}}{c_p}\right)\left(\frac{p_0}{p}\right)^{\kappa}. \quad (4.1)$$

$$\theta = T\,(p_0 / p)^{R/cp}$$

$$\omega = \frac{\partial\theta/\partial t + \mathbf{V} \cdot \nabla_p \theta}{-\partial\theta/\partial p} \quad (4.2)$$

$$\omega_\theta = \left(\frac{\partial p}{\partial t}\right)_\theta + (\mathbf{V}_\theta \cdot \nabla_\theta p), \quad (4.3)$$

$$v_{g\theta} = \frac{1}{f_0}\frac{\partial M}{\partial x} \quad (4.4)$$

$$u_{g\theta} = -\frac{1}{f_0}\frac{\partial M}{\partial y} \quad (4.5)$$

$$M = (c_p\,T \; + \; g\;Z)_\theta \quad (4.6)$$

$$\frac{d}{dt}[(f + \zeta_\theta)(\frac{\partial\theta}{\partial p})] = 0. \quad (4.7)$$

Formula Sheet 9 for Exam#3 **ATMS 410**

$$\frac{\delta S}{\delta t} = \frac{\partial S}{\partial t} + \mathbf{C} \cdot \nabla S \tag{4.8}$$

$$\left(\frac{\partial S}{\partial t}\right)_\theta = -\mathbf{C}_\theta \cdot \nabla_\theta S. \tag{4.9}$$

$$\left(\frac{\partial p}{\partial t}\right)_\theta = -C_{\theta x}\left(\frac{\partial p}{\partial x}\right)_\theta . \tag{4.10}$$

$$\omega_\theta = \mathbf{V}_{\mathbf{R}\theta} \cdot \nabla_\theta p \approx (\mathbf{V}_\mathbf{R} \cdot \nabla_p \theta)/(-\frac{\partial \theta}{\partial p}) \tag{4.11}$$

$$\mathbf{V}_{\mathbf{R}\theta} = \mathbf{V}_\theta - C_x \mathbf{i} - C_y \mathbf{j} \tag{4.12}$$

$$\zeta = -\frac{\partial V}{\partial n} + \frac{V}{R_S}$$

$$0 = -\frac{V^2}{r_T} - g\frac{\partial z}{\partial n} - fV$$