Formula Sheet 1 for Exam#2 **ATMS 410**

$$\frac{du}{dt} = fv - \frac{1}{\rho}\frac{\partial p}{\partial x} + F_x \tag{2.1}$$

$$\frac{dv}{dt} = -fu - \frac{1}{\rho}\frac{\partial p}{\partial y} + F_y \tag{2.2}$$

$$\frac{dw}{dt} = -fu\cot\phi - \frac{1}{\rho}\frac{\partial p}{\partial z} - g + F_z \tag{2.3}$$

$$v_g = \frac{1}{f\rho}\frac{\partial p}{\partial x} \tag{2.4}$$

$$u_g = -\frac{1}{f\rho}\frac{\partial p}{\partial y} \tag{2.5}$$

$$v_g = \frac{g}{f}\left(\frac{\partial Z}{\partial x}\right)_p \tag{2.6}$$

$$u_g = -\frac{g}{f}\left(\frac{\partial Z}{\partial y}\right)_p \tag{2.7}$$

$$\nabla_p \cdot \mathbf{V} = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = -\frac{\partial \omega}{\partial p} = D_p \tag{2.8}$$

$$D_{pg} = -(\beta/f)v_g \tag{2.9}$$

$$\mathbf{V} = \mathbf{V}_\psi + \mathbf{V}_\chi \tag{2.11}$$

$$\nabla \cdot \mathrm{V}_\psi = 0 \qquad \nabla \times \mathrm{V}_\chi = 0$$

$$\mathbf{V}_\psi = \mathbf{k}\times\nabla\psi \tag{2.12}$$

$$u_\psi = -\frac{\partial \psi}{\partial y}, \quad v_\psi = \frac{\partial \psi}{\partial x} \tag{2.13}$$

$$\zeta = \mathbf{k}\cdot\nabla\times\mathbf{V} = \nabla^2\psi. \tag{2.14}$$

$$\nabla^2\chi = -\nabla\cdot\mathbf{V} \tag{2.15}$$

$$u_\chi = -\frac{\partial\chi}{\partial x}, \quad v_\chi = -\frac{\partial\chi}{\partial y} \tag{2.16}$$

$$\frac{\partial p}{\partial z} = -\rho g \tag{2.17}$$

$$\frac{du}{dt} = fv - g\frac{\partial Z}{\partial x} + F_x = f\left[v - v_g\right] + F_x = f v_{ag} + F_x$$

$$\frac{dv}{dt} = -fu - g\frac{\partial Z}{\partial y} + F_y = -f\left[u - u_g\right] + F_y = -f u_{ag} + F_y$$

$$\mathbf{V_T} = \frac{\partial \mathbf{V_g}}{\partial \ln p} = -\hat{k}\times\frac{R}{f}\frac{\partial \overline{T}_v}{\partial n} \tag{2.18}$$

$$\frac{\partial(\zeta+f)}{\partial t} = -\mathbf{V}\cdot\nabla_p(\zeta+f) - \omega\frac{\partial\zeta}{\partial p} - (\zeta+f)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) - \left(\frac{\partial\omega}{\partial x}\frac{\partial v}{\partial p} - \frac{\partial u}{\partial p}\frac{\partial\omega}{\partial y}\right) + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right). \tag{2.19}$$

$$\frac{\partial\zeta_g}{\partial t} = -\mathbf{V}_g\cdot\nabla_p\eta_g - f_0\nabla_p\cdot\mathbf{V} + f_0\mathbf{k}\cdot(\nabla_p\times\mathbf{F}) \tag{2.20}$$

$$\zeta_g = \frac{\nabla_p^2\Phi}{f_0}, \quad \text{where} \quad Z \equiv \frac{\Phi(z)}{g_0},\ g_0 = 9.81\,\mathrm{m\,s^{-2}} \qquad \eta_g = \zeta_g + f = \frac{\nabla_p^2\Phi}{f_0} + f$$

$$\frac{\partial}{\partial t}\left(-\frac{\partial\Phi}{\partial p}\right) = -\mathbf{V}_g\cdot\nabla_p\left(-\frac{\partial\Phi}{\partial p}\right) + \sigma\omega + \frac{\dot{Q}}{c_p\rho} \tag{2.21}$$

$$\frac{\partial T}{\partial t} = -\mathbf{V}_g\cdot\nabla_p T + \frac{\omega}{c_p\rho} + \frac{\dot{Q}}{c_p} \tag{2.22}$$

Formula Sheet 3 for Exam#2 **ATMS 410**

$$Z_t - Z_b = \frac{R\overline{T_v}}{g_0} ln\left(\frac{p_b}{p_t}\right), \tag{3.1}$$

$$p_h = \int_h^\infty g\rho dz. \tag{3.2}$$

$$\frac{\partial p_s}{\partial t} \approx -\int_0^{p_s} \nabla_p \cdot \mathbf{V}\ dp, \tag{3.3}$$

$$\frac{\delta p_s}{\delta t} = \frac{\partial p_s}{\partial t} + \mathbf{C} \cdot \nabla_z p_s \tag{3.4}$$

$$\frac{\partial p_s}{\partial t} = -\mathbf{C} \cdot \nabla_z p_s. \tag{3.5}$$

$$\begin{aligned}\left(\nabla_p^2 + \frac{{f_0}^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega &= \frac{f_0}{\sigma}\frac{\partial}{\partial p}\left(\mathbf{V}_g \cdot \nabla_p\left(\frac{\nabla_p^2\Phi}{f_0} + f\right)\right) \\ &+ \frac{1}{\sigma}\nabla_p^2\left(\mathbf{V}_g \cdot \nabla_p\left(-\frac{\partial\Phi}{\partial p}\right)\right) \\ &- \frac{1}{\sigma}\nabla_p^2\left(\frac{\dot{Q}}{\rho c_p}\right) \\ &- \frac{f_0}{\sigma}\frac{\partial}{\partial p}(\mathbf{k} \cdot \nabla_p \times \mathbf{F}).\end{aligned} \tag{3.6}$$

$$\omega(x,y,p) = \omega_0 \sin\left(\frac{2\pi x}{L}\right)\sin\left(\frac{2\pi y}{L}\right)\left(\frac{(p_0 - p)(p - p_t)}{(\frac{1}{2})^2(p_0 - p_t)^2}\right) \tag{3.7}$$

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = -\left(\omega_v + \omega_T + \omega_H + \omega_f\right) \tag{3.8}$$

$$\omega_v = \frac{f_0}{\sigma}\frac{\partial}{\partial p}\left(-\mathbf{V}_g \cdot \nabla_p\left(\frac{\nabla_p^2\Phi}{f_0} + f\right)\right) = \frac{f_0}{\sigma}\frac{\partial}{\partial p}(-\mathbf{V}_g \cdot \nabla_p \eta_g). \tag{3.13}$$

$$\omega_T = \frac{-1}{\sigma}\nabla_p^2\left(\mathbf{V}_g \cdot \nabla_p\left(-\frac{\partial\Phi}{\partial p}\right)\right) = \frac{R}{\sigma p}\nabla_p^2(-\mathbf{V}_g \cdot \nabla_p T) \tag{3.17}$$

$$\left(\nabla_p^2 + \frac{{f_0}^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = \frac{2}{\sigma}\frac{\partial \mathbf{V}_g}{\partial p} \cdot \nabla_p \eta_g. \tag{3.19}$$

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\frac{\partial \Phi}{\partial t} = -f_0 \mathbf{V}_g \cdot \nabla_p \eta_g + \frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left(\mathbf{V}_g \cdot \nabla_p \left(-\frac{\partial \Phi}{\partial p}\right)\right)$$

$$-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left(\frac{\dot{Q}}{\rho c_p}\right) \quad (3.20)$$

$$\sigma = \frac{1}{\theta}\frac{\partial \Phi}{\partial p}\frac{\partial \theta}{\partial p} = g\frac{\partial z}{\partial p}\left(\frac{1}{T}\frac{\partial T}{\partial p} - \frac{\kappa}{p}\frac{\partial p}{\partial p}\right) = \frac{R}{p}\left(\Gamma_d - \frac{\partial T}{\partial p}\right) \quad (3.21)$$

$$\left(-2\left(\frac{2\pi}{L}\right)^2 \sigma + \left(\frac{-8 f_0^2}{(p_0 - p_T)^2}\right)\right)\omega_d = -forcing = -\sigma(\omega_v + \omega_T + \omega_H + \omega_f) \quad (3.25)$$

which can be written in shorthand as

$$(A_d + B)\omega_d = +forcing \quad (3.26)$$

$$A_d = 2\left(\frac{2\pi}{L}\right)^2 \sigma = 2\left(\frac{2\pi}{L}\right)^2 \frac{R}{p}\left(\Gamma_d - \frac{\partial T}{\partial p}\right) \quad (3.27)$$

and

$$B = \frac{8 f_0^2}{(p_0 - p_T)^2}. \quad (3.28)$$

$$\frac{\omega_N}{\omega_d} = \frac{A_d + B}{B} \quad (3.33)$$

$$\sigma \equiv \frac{R}{p}\left(\left(\frac{dT}{dp}\right)_m - \frac{\partial T}{\partial p}\right) \quad (3.36)$$

$$\omega_m = \frac{forcing}{A_m + B} \quad (3.37)$$

$$A_m = 2\left(\frac{2\pi}{L}\right)^2 \sigma = 2\left(\frac{2\pi}{L}\right)^2 \frac{R}{p}\left(\Gamma_m - \frac{\partial T}{\partial p}\right) \quad (3.38)$$

$$\left(\frac{\partial \zeta_g}{\partial t}\right)_{p_0} = [-\mathbf{V}_g \cdot \nabla_p \eta_g]_{p_{ND}} + \frac{\nabla_p^2}{f_0}\int_{p_{ND}}^{p_0}\left(\frac{R}{P}(\mathbf{V}_g \cdot \nabla_p T) - \sigma\omega - \frac{\dot{Q}}{c_p \rho}\right) dp \quad (3.46)$$

$$\left(\frac{\partial \zeta_g}{\partial t}\right)_{p_0} = [-\mathbf{V}_g \cdot \nabla_p \eta_g]_{500} - \frac{g}{f}\nabla_p^2\left(-\mathbf{V}_{g_0} \cdot \nabla_p (Z_{500} - Z_{1000}) + \bar{\sigma\omega} + \frac{\bar{\dot{Q}}}{c_p \rho}\right) \quad (3.47)$$

$$\frac{\partial(\zeta + f)}{\partial t} = -(\zeta + f)\nabla_{\mathbf{p}} \cdot \mathbf{V}, \quad (3.49)$$

$$\zeta = -\frac{\partial V}{\partial n} + \frac{V}{R_S}$$

$$0 = -\frac{V^2}{r_T} - g\frac{\partial z}{\partial n} - fV$$