Formula Sheet 1 for Exam#1 **ATMS 410**

$$[x] = \frac{1}{2\pi a \cos\phi} \int_0^{2\pi} x a \cos\phi d\lambda \qquad (1.1)$$

$$\overline{x} = \frac{1}{T} \int_0^T x dt \qquad (1.2)$$

$$[inflow + sources] - [outflow + sinks] - [storage] = 0 \qquad (1.3)$$

$$M = a\cos\phi(\Omega a \cos\phi + u) \qquad (1.4)$$

$$u_f = \frac{\Omega a^2 (\cos^2\phi_i - \cos^2\phi_f) + u_i(a\cos\phi_i)}{(a\cos\phi_f)} \qquad (1.5)$$

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} - \omega\frac{\partial T}{\partial p} + \gamma_d\omega + \frac{\dot{Q}}{c_p} \qquad (1.6)$$

$$\frac{\partial T}{\partial t} = -\frac{\partial uT}{\partial x} - \frac{\partial vT}{\partial y} - \frac{\partial \omega T}{\partial p} + \gamma_d\omega + \frac{\dot{Q}}{c_p}. \qquad (1.7)$$

$$uT = ([u] + u^*)([T] + T^*) = [u][T] + u^*[T] + [u]T^* + u^*T^* \qquad (1.13)$$

$$[uT] = [[u][T]] + [u^*[T]] + [[u]T^*] + [u^*T^*] = [u][T] + [u^*T^*]. \qquad (1.14)$$

$$\frac{\partial [T]}{\partial t} = -\frac{\partial [v][T]}{\partial y} - \frac{\partial [\omega][T]}{\partial p} + \gamma_d[\omega] + [\frac{\dot{Q}}{c_p}] - \frac{\partial [v^*T^*]}{\partial y} - \frac{\partial [\omega^*T^*]}{\partial p} \qquad (1.15)$$

$$\frac{\partial [u]}{\partial t} = -\frac{1}{\cos^2\phi}\frac{\partial [u][v]\cos^2\phi}{\partial y} - \frac{\partial [u][\omega]}{\partial p} + f[v] + [F_x] - \frac{1}{\cos^2\phi}\frac{\partial [u^*v^*]\cos^2\phi}{\partial y} - \frac{\partial [u^*\omega^*]}{\partial p} \qquad (1.16)$$

$$\overline{uT} = \overline{(\overline{u} + u')(\overline{T} + T')} = \overline{u}\overline{T} + \overline{u'\overline{T}} + \overline{\overline{u}T'} + \overline{u'T'} \qquad (1.17)$$

$$\overline{uT} = \overline{(\overline{u} + u')(\overline{T} + T')} = \overline{u}\overline{T} + \overline{u'T'} \qquad (1.18)$$

$$\overline{[v][T]} = \overline{[v]}\ \overline{[T]} + \overline{[v]'[T]'} \qquad (1.21)$$

and

$$\overline{[v^*T^*]} = [\overline{v^*}\ \overline{T^*}] + [\overline{v^{*\prime}T^{*\prime}}] \qquad (1.22)$$

$$\begin{aligned} 0 = \frac{\partial\overline{[T]}}{\partial t} = &-\frac{\partial\overline{[v]}\ \overline{[T]}}{\partial y} - \frac{\partial\overline{[\omega]}\ \overline{[T]}}{\partial p} + \gamma_d\overline{[\omega]} + \frac{\overline{[\dot{Q}]}}{c_p} \\ &-\frac{\partial\overline{[v]'[T]'}}{\partial y} - \frac{\partial\overline{[\omega]'[T]'}}{\partial p} \\ &-\frac{\partial[\overline{v^*T^*}]}{\partial y} - \frac{\partial[\overline{\omega^*T^*}]}{\partial p} \\ &-\frac{\partial[\overline{v^{*\prime}T^{*\prime}}]}{\partial y} - \frac{\partial[\overline{\omega^{*\prime}T^{*\prime}}]}{\partial p} \qquad (1.23) \end{aligned}$$

$$\begin{aligned} 0 = \frac{\partial\overline{[u]}}{\partial t} = &-\frac{1}{\cos^2\phi}\frac{\partial\overline{[u]}\ \overline{[v]}\cos^2\phi}{\partial y} - \frac{\partial\overline{[u]}\ \overline{[\omega]}}{\partial p} + f\overline{[v]} + \overline{[F_x]} \\ &-\frac{1}{\cos^2\phi}\frac{\partial\overline{[u]'[v]'}\cos^2\phi}{\partial y} - \frac{\partial\overline{[u]'[\omega]'}}{\partial p} \\ &-\frac{1}{\cos^2\phi}\frac{\partial[\overline{u^*}\ \overline{v^*}]\cos^2\phi}{\partial y} - \frac{\partial[\overline{u^*}\ \overline{\omega^*}]}{\partial p} \\ &-\frac{1}{\cos^2\phi}\frac{\partial[\overline{u^{*\prime}v^{*\prime}}]\cos^2\phi}{\partial y} - \frac{\partial[\overline{u^{*\prime}\omega^{*\prime}}]}{\partial p} \qquad (1.24) \end{aligned}$$

Formula Sheet 3 for Exam#1 **ATMS 410**

$$\text{NAO index} = \text{SLP}'_{AZ} - \text{SLP}'_{ICE}$$

$$\text{PNA index} = \frac{Z'_{(1)} - Z'_{(2)} + Z'_{(3)} - Z'_{(4)}}{4}$$

$$\bar{P} \propto \frac{1}{V}\int \frac{\overline{\theta'^2}}{\bar{\theta}^2}\, dV$$

$$E_\text{I} = c_v \int_0^\infty \rho T\, dz$$

$$E_\text{p} = \int_0^\infty p\, dz = R\int_0^\infty \rho T\, dz$$

$$E_\text{p} + E_\text{I} = \frac{c_p}{c_v} E_\text{I} = \frac{c_p}{R} E_\text{p}$$

$$(K_E, K_Z) = \int_m [u_g^* v_g^*] \frac{\partial [u_g]}{\partial y} dm \qquad (1.25)$$

$$(A_Z, K_Z) = -R_d \int_m \frac{[\omega][T]}{p} dm \qquad (1.26)$$

$$(D_Z) = \int_m [u_g][F_x] dm \qquad (1.27)$$

$$(A_E, K_E) = -R_d \int_m \frac{[\omega^* T^*]}{p} dm \qquad (1.28)$$

$$(D_E) = \int_m \left([\mathbf{V}_g^* \cdot \mathbf{F}^*]\right) dm \qquad (1.29)$$

$$(A_E, A_Z) = R_d \int_m \frac{1}{\sigma}\left([v_g^* T^*]\right) \frac{\partial [T]}{\partial y} dm \qquad (1.30)$$

$$(G_Z) = \frac{R_d}{c_p}\int_m \frac{[\dot{Q}_{db}][T]}{\sigma p} dm \qquad (1.31)$$

$$(G_E) = \frac{R_d}{c_p}\int_m \frac{[\dot{Q}^*_{db} T^*]}{\sigma p} dm \qquad (1.32)$$