The natural coordinate system is another way of representing direction. It is based on the relative motion of the object of interest, rather than a fixed coordinate plane like height (x,y,z,t) or isobaric (x,y,p,t) coordinates.

The unit vectors are:

- **t** - Oriented parallel to the horizontal velocity at each point
- **n** - Oriented perpendicular to the horizontal velocity and pointing positively to the left of the flow direction
- **k** - Directed vertically upward

We also define ‘s’ as the horizontal distance along a curve followed by the air parcel, and ‘R’ is the radius of curvature following the parcel motion. When the center of curvature is in the positive n direction (as below), R>0:

![Diagram of natural coordinates](image)

The **acceleration** of the parcel is the sum of the change in speed along its path and the centripetal acceleration:

\[
\frac{D\vec{V}}{Dt} = \vec{t} \frac{D\vec{V}_H}{Dt} + \vec{n} \frac{V^2_H}{R}
\]

where:

a) Total rate of change of velocity with time (acceleration)
b) Change in the horizontal speed with time in the t direction
c) Centripetal acceleration (pointing in the positive n direction)

The **Coriolis Force** in natural coordinates is a simple transformation since it always acts normal to the direction of the flow:

\[-f\vec{k} \times \vec{V} = -fV_H \vec{n}\]
The **Pressure Gradient Force** in natural coordinates (on a constant pressure surface) can be expressed as:

\[- \nabla_p \Phi = - \left( \frac{\partial \Phi}{\partial s} + \hat{n} \frac{\partial \Phi}{\partial n} \right) \]

where:

a) How geopotential changes along the direction of motion
b) How geopotential changes perpendicular to direction of motion

Motion in the horizontal plane (**horizontal momentum equation**) can be expanded into parallel and normal components to the direction of flow:

\[
\begin{align*}
\frac{DV_H}{Dt} &= - \frac{\partial \Phi}{\partial s} \\
\frac{V_H^2}{R} + fV_H &= - \frac{\partial \Phi}{\partial n}
\end{align*}
\]

The top expression is the change in horizontal velocity along the direction of flow (s-direction), and the bottom expression is the net forces acting normal to the flow.

Note that \( \frac{\partial \Phi}{\partial s} = 0 \) for motion parallel to the geopotential height contours and thus the speed is constant following the motion.

For the bottom expression,

a) Centrifugal acceleration
b) Coriolis Force
c) Pressure gradient force (normal to flow)

Flow can be classified into various categories based on the relative contributions of these three terms. This is discussed in the next section, “Balanced Flow”.

\[ \frac{\partial \Phi}{\partial n} < 0 \]