Geopotential

Definition: Work that must be done against the Earth's gravitational field in order to raise a mass of 1 kg from sea level to that point.

Units: 'Work' is defined as force * distance. It has units of Joules (J). So the work to lift a mass a certain height is:

$$J\ kg^{-1}$$

Since a Joule is also defined as a Newton * meter, where Newton = kg m s^{-2}, we can substitute for J above to yield an alternative unit for geopotential:

$$m^2\ s^{-2}$$

The symbol for geopotential is Φ. Geopotential is defined as having a magnitude of 0 at the Earth's surface. The Φ at any height above the Earth's surface is equivalent to the distance traveled multiplied by the gravity at each integration of height:

$$\Phi(z) = \int_0^z g\,dz \qquad (1)$$

where z = height above the surface, and g is the gravity at each height. Note that gravity is not constant throughout the atmosphere – it decreases as the distance from the center of the earth increases. So it is not something that can be pulled outside the integral.

The ***geopotential height*** is defined as the geopotential at height z (Equation 1 above) divided by the gravitational acceleration at the surface of the Earth (g_o). It is represented as a capital 'Z'.

$$Z \equiv \frac{\Phi(z)}{g_o} = \frac{1}{g_o}\int_0^z g\,dz \qquad (2)$$

g_o is a constant (9.8 ms^{-2}). In the lower atmosphere, Z is very close to z (called the 'geometric height'). The table below shows how Z,z, and g vary with height at a typical mid-latitude location (from Wallace and Hobbs p. 55):

z(km)	Z(km)	g(ms^{-2})
0	0	9.802
1	1.000	9.798
10	9.986	9.771
20	19.941	9.741
30	29.864	9.710
60	59.449	9.620

90	88.758	9.531
120	117.795	9.443
160	156.096	9.327
200	193.928	9.214
300	286.520	8.940
400	376.370	8.677
500	463.597	8.427
600	548.314	8.186

Note that as geometric altitude (z) increases, Z becomes increasingly less because the gravitational acceleration is decreasing. This means less work is required ($\Phi(z)$ is getting smaller) to lift the mass to that point because the opposing force (g) is decreasing.

So why care about Z and Φ? Well, there are several reasons.

1) It can be used to derive the hypsometric equation, which tells us that the thickness of an atmospheric layer is proportional to its mean virtual temperature. You did part of this on your homework assignment, substituting the equation of state into the hydrostatic equation and going from there.

2) Effective gravity (g) is perpendicular to lines of constant geopotential surfaces, not geometric height

3) It is frequently used to convert surface pressures to mean sea level pressures, through manipulation of the hypsometric equation. See Wallace and Hobbs pages 59-60 for an explanation.