

Fundamental Forces

Two categories of forces:

- 1) **Body** forces – Forces whose magnitudes are proportional to the mass of the body being acted upon. Usually treated as constants, even though they may change slightly as the distance between interacting bodies increases
Example – Gravitational force
- 2) **Surface** forces – Result from molecular (small-scale) interactions along surface elements. Independent of mass.
Example – Pressure gradient force, friction force

Fundamental Forces influence atmospheric motions in a fixed coordinate system (one that isn't moving)

Newton's laws of motion provide the basis for atmospheric motion:

- 1) A body will continue in its state of rest or uniform straight line motion unless it is acted upon by a force
- 2) The magnitude of a force acting upon the body is given by:

$$F = ma \quad (1)$$

Where m = mass of the body (kg), and a = acceleration (ms^{-2})

- 3) To every action there is an opposite and equal reaction

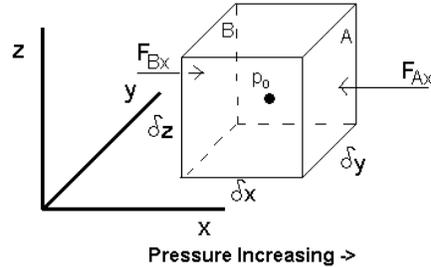
We are particularly interested in law #2. The forces that act on atmospheric bodies (air molecules) are:

- 1) Pressure Gradient Force
- 2) Gravitational Force
- 3) Friction Force

Note that these forces are for motion relative to a *fixed* coordinate plane. In the real atmosphere, motions are occurring relative to a moving coordinate plane (earth rotation). The next section will deal with additional (*apparent*) forces that we must consider in that situation.

Pressure Gradient Force

Consider a small **air parcel** (volume of air) located in an area where the pressure is increasing from left to right:



First, we would like to find the pressure force exerted on face A of the block (F_{Ax}). We know that the force exerted from the outside of face A must equal the force exerted from inside the parcel because the parcel is neither expanding nor contracting in that direction. Thus,

$$F_{Ax} = - \left(p_0 + \frac{\partial p}{\partial x} \frac{\delta x}{2} \right) \delta y \delta z$$

In plain English, the pressure force from the outside of face A is equal to the pressure at point p_0 (at the center of the box) plus the change in pressure in the x-direction ($\frac{\partial p}{\partial x}$) over a distance of half the box length multiplied by the area of face A ($\delta y \delta z$). The negative sign indicates that the pressure force F_{Ax} is directed in the negative x direction.

Similarly, we can write the pressure force acting on face B as:

$$F_{Bx} = + \left(p_0 - \frac{\partial p}{\partial x} \frac{\delta x}{2} \right) \delta y \delta z$$

The net horizontal force on the parcel is found by adding F_{Ax} and F_{Bx} :

$$F_x = F_{Ax} + F_{Bx} = - \frac{\partial p}{\partial x} \delta x \delta y \delta z \quad (2)$$

The net force is called the **pressure gradient force** (PGF) because it is proportional to the derivative of pressure in that direction ($\frac{\partial p}{\partial x}$). To derive the PGF per unit mass, note that mass = density x volume:

$$m = \rho \delta x \delta y \delta z$$

Divide Eq. 2 by the mass of the parcel to yield the x-component of the PGF:

$$\frac{F_x}{m} = -\frac{1}{\rho} \frac{\partial p}{\partial x} \quad (3)$$

Similar expressions can be found for the PGF in the y and z directions. The total PGF per unit mass in 3-D is:

$$\frac{F_{xyz}}{m} = -\frac{1}{\rho} \nabla p \quad (4)$$

Gravitational Force

Newton's law of universal gravitation says two masses will attract each other with a force given by:

$$F_g = -\frac{GMm}{r^2}$$

The force per unit mass is:

$$\frac{F_g}{m} = g_o = -\frac{GM}{r^2} = \text{Gravitational Acceleration}$$

Where G = gravitational constant ($6.673 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$), M = mass of earth, and r = distance between the earth and the object it is imparting its gravitational attraction to.

For this course, the radius of the earth ($6.37 \times 10^6 \text{ m}$) \gg any height above mean sea level (MSL) with significant atmospheric motion. So although the gravitational force does decrease with height above the surface, we will treat it as a constant (9.8 ms^{-2}).

Viscous (Friction) Force

Molecules in the atmosphere are subject to internal friction, causing it to resist the tendency to flow. We'll discuss more about this later in the course. For now, the friction force per unit mass is:

$$\mathbf{F}_r / m = F_{rx}\mathbf{i} + F_{ry}\mathbf{j} + F_{rz}\mathbf{k}$$

The frictional force represents the collective effects of all scales of motion in the equations of motion. Throughout most of the atmosphere friction is so small that it can be safely neglected. However, motions in the [planetary boundary layer](#) (generally $< 1 \text{ km}$ in altitude) experience a frictional drag force that is comparable in magnitude to other terms in the equations of motion. So it must be included.