Balanced Flow
Geostrophic, Inertial, Gradient, and Cyclostrophic Flow

The types of atmospheric flows described here have the following characteristics:

1) Steady state (meaning that the flows do not change with time)
2) No vertical velocity component
3) Natural coordinate system (constant pressure surfaces)

These are “idealized” flows, created by balances of horizontal forces. They provide a qualitative way of understanding several types of flows in the atmosphere.

Geostrophic Flow

We have already discussed geostrophic flow in (x,y,z) coordinates. Recall that geostrophic flow results from a balance of the pressure gradient and Coriolis Forces. It is straight line, non-accelerating flow parallel to the isobars on a constant height surface.

\[-f_{vg} = -\alpha \frac{\partial p}{\partial x}\] (1)

\[f_{ug} = -\alpha \frac{\partial p}{\partial y}\] (2)

Recall from the previous lecture on natural coordinates the expression for the force balances normal to the direction of flow:

\[\frac{V_{H \perp}^2}{R} + fV_{H \perp} = -\frac{\partial \Phi}{\partial n}\] (3)

Thus, geostrophic balance in natural coordinates is:

\[fV_{g} = -\frac{\partial \Phi}{\partial n}\] (4)

since there is no centrifugal force \((V^2/R)\) in straight line flow. Geostrophic flow in natural coordinates is parallel to the geopotential lines:
Inertial Flow

If the geopotential field is uniform across the region, then \( \frac{\partial \Phi}{\partial n} = 0 \). Equation (3) becomes:

\[
\frac{V_H^2}{R} + fV_H = 0
\]  

(5)

Solving for the radius of curvature (R):

\[
R = \frac{-V}{f}
\]  

(6)

On a uniform geopotential field, velocity cannot change. How do we know this? From the natural coordinates lecture, we saw that the change in velocity along the direction of the flow was due to the change of the geopotential field along that path:

\[
\frac{DV_H}{Dt} = -\frac{\partial \Phi}{\partial s}
\]

For a uniform geopotential field, \( \frac{DV_H}{Dt} = 0 \). Getting back to Equation (6), if we are at a constant latitude (f not changing), the radius of curvature must also be constant. So any air parcels that are moving will flow in a circular fashion, anti-cyclonically in the Northern Hemisphere. We know it must be anti-cyclonic motion since the Coriolis Force pulls air to the right in the NH.

This type of motion is called an \textit{inertial oscillation} since the motion of the parcel itself is what gives rise to the Coriolis and Centrifugal forces. In the atmosphere, inertial motion...
is almost never observed since virtually all motion is produced by pressure gradient forces. However, inertial motions are common in the oceans, where pressure gradients frequently do not exist and motion may be induced by wind flows at the surface.

**Cyclostrophic Flow**

For very small horizontal scales, the Coriolis Force can be neglected. This approximation is called cyclostrophic flow:

\[
\frac{V_H^2}{R} = -\frac{\partial \Phi}{\partial n} \tag{7}
\]

The cyclostrophic wind can be found by solving for \(V\):

\[
V = \left(-R \frac{\partial \Phi}{\partial n}\right)^{\frac{1}{2}} \tag{8}
\]

Cyclostrophic flow can be either cyclonic (counter-clockwise) or anti-cyclonic (clockwise). Note that the PGF always acts towards the center of rotation:

As in the geostrophic approximation, the Rossby Number \((R_o)\) can be used to determine whether it is OK to apply cyclostrophic flow. Remember that the Rossby number is a ratio of the horizontal wind to the Coriolis acceleration. For cyclostrophic flow, we would like the Rossby number to be very large, which would indicate that the Coriolis accelerations can be neglected. The Rossby number is:

\[
R_o = \frac{U}{f_o L} \tag{9}
\]
Let’s use a tornado in the mid-latitudes as an example. If the tangential velocity is 30 m/s at a radius (L) or 300 m from the center, and \( f = 10^{-4} \, \text{s}^{-1} \), the Rossby number \( \approx 10^3 \). So we can conclude that we can apply cyclostrophic flow to describe tornadic circulations. And indeed, tornadoes have been observed to rotate both cyclonically and anticyclonically (although a cyclonic rotation is preferred).

**Gradient Wind Approximation**

**Gradient Flow** refers to the situation in which the flow is horizontal, frictionless, and parallel to the height contours. Thus,

\[
\frac{DV}{Dt} = -\frac{\partial \Phi}{\partial s} = 0
\]

So we have a 3-way balance between the horizontal pressure gradient force, Coriolis Force, and centrifugal force:

\[
\frac{V_H^2}{R} + fV_H = -\frac{\partial \Phi}{\partial n}
\]  

Equation (10) is commonly called the **Gradient Wind Equation**. It is considered a better wind approximation than the geostrophic approximation since it takes into account curved flow.

Equation (10) can be solved for \( V_{gr} \), the gradient wind:

\[
V_{gr} = -\frac{fR}{2} \pm \left( \frac{f^2 R^2}{4} + fRV_g \right)^{1/2}
\]

where \( V_g \) is the geostrophic wind. There are four realistic solutions to Equation (11), summarized in the table below (Table 3.1 in Holton):
And these are graphically illustrated in Fig. 3.5 in Holton:

- The upper left hand corner of the above figure (a) represents a “normal low” situation, where the PGF = Co+Ce.

- The upper right hand corner (b) represents a “normal high” situation, where the outward pointing PGF and Centrifugal force (Ce) is balanced by the inward pointing Co.
- The lower left hand corner (c) represents an “anomalous low” situation, anomalous because the Coriolis Force and PGF point in the same direction. This situation is called antibaric (a baric case is one in which the PGF and CF oppose each other, as normal). It is thought that this situation is unrealistic for larger scale flows in the atmosphere, and only applicable for small-scale intense vortices (e.g. tornadoes).

- The lower right hand corner (d) represents an “anomalous high”.